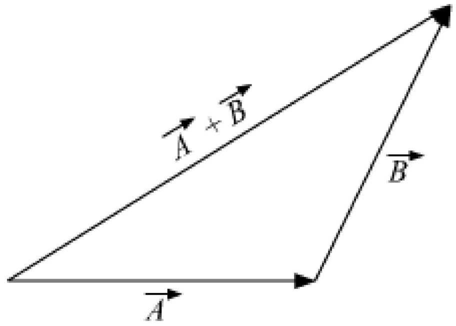
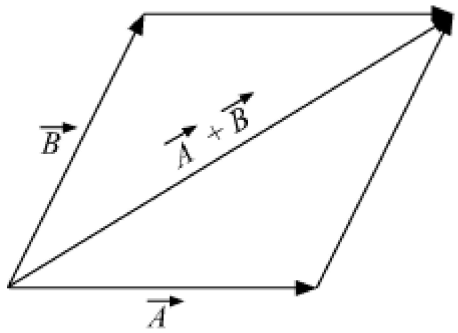
2. Scalars and Vectors

- **Scalar quantities:** These are the physical quantities that are not affected by the change in the coordinate systems used to define them. They do not have any direction. Example: Speed, charge, temperature, etc.
- **Vector quantities:** These are physical quantities that have both direction and magnitude. They change with change in the coordinate systems used to define them. Example: Displacement, velocity, etc.
- **Position vector:** Position vector of a point in a coordinate system is the straight line that joins the origin and the point.
- **Displacement Vector:** It is the straight line that joins the initial and the final position.
- **Equality of Vectors:** Two vectors are said to be equal only if they have the same magnitude and the same direction.
- **Negative vector:** Negative vector is a vector whose magnitude is the same as that of a given vector, but whose direction is opposite to that of the given vector.
- **Zero vector:** Zero vector is a vector whose magnitude is zero and have an arbitrary direction.
- **Resultant vector:** The resultant vector of two or more vectors is the vector which produces the same effect as produced by the individual vectors together.
- **Multiplication of Vectors by Real Numbers**
    - Multiplication of a vector with a positive number k only changes the magnitude of the vector keeping its direction unchanged.
    - Multiplication of a vector with a negative number $-k$ changes the magnitude and direction of the vector.
- **Addition of vectors:**
    - Head-to-tail/ triangle method



- Parallelogram method

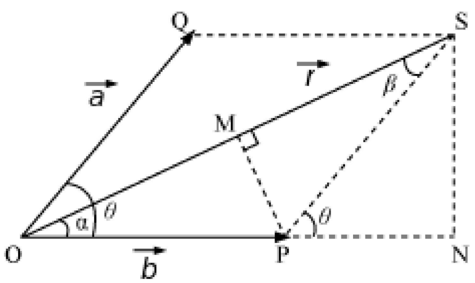


- **Vector addition follows commutative and associative laws**:
  - $\vec{A}+\vec{B}=\vec{B}+\vec{A}$ [Commutative]
  - $\vec{A}+\vec{B}+\vec{C}=\vec{A}+\vec{B}+\vec{C}$ [Associative]
- **Subtraction of vector:**

  $$\vec{A}-\vec{B}=\vec{A}+-\vec{B}$$

- **Polygon law of vector addition:**
  - According to this law, if a number of vectors acting in a plane are represented in magnitudes and directions by the sides of an open polygon taken in order, then resultant vector is represented in magnitude and direction by the closing side of the polygon taken in the opposite order. The direction of the resultant vector is from the starting point of the first vector to the end point of the last vector.

- **Resultant of two vectors**



$$\vec{r}=\vec{a}+\vec{b}$$

- **Law of cosines**

  $$\vec{r}=\vec{a}+\vec{b}+2\vec{a}\vec{b}\cos\theta$$

- **Law of sines**

  $$\frac{|r|}{\sin\theta}=\frac{|a|}{\sin\beta}=\frac{|b|}{\sin\alpha}$$

- **Unit vector:** Unit vector is a vector of unit magnitude along the direction of the vector.

  $$\hat{a}=\vec{a}a, \quad \vec{a}=\hat{a}a$$

- In 2-D vector, $\vec{a}$ can be expressed as $\vec{a}=a_x\hat{i}+a_y\hat{j}$
- If $\vec{a}$ makes θ angle with X axis, then

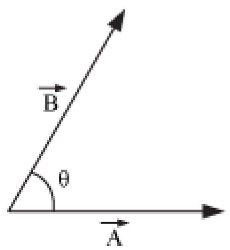
$a_x = a \cos\theta$ and $a_y = a \sin\theta$

$$a = |a| = \sqrt{a_x^2 + a_y^2},\ \tan\theta = \frac{a_y}{a_x}$$

- The same process is used to resolve a vector into three components along X-axis, Y-axis, and Z-axis.

- Scalar product of two vectors $\vec{A}$ and $\vec{B}$ is given by

$$\vec{A}.\vec{B} = AB\cos\theta$$



- The result of the scalar product of two vectors is a scalar quantity.
- When two vectors are parallel their scalar product is equal to the product of their magnitudes.
- When two vectors are perpendicular their scalar product is equal to zero.
- **Properties of Scalar Product of two vectors**
  - Scalar product of two vectors is commutative, i.e.,

$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

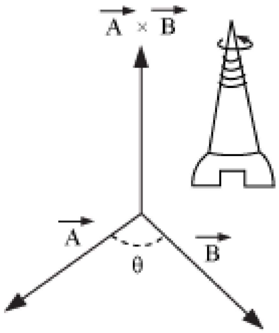
  - Scalar product is distributive, i.e.,

$$\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$$

- Scalar product of a vector with itself gives the square of its magnitude.
- Dot Product of two vectors $\vec{A}$ and $\vec{B}$ in Cartesian Coordinates is

$$\vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

- The magnitude of the vector product of two vectors $\vec{A}$ and $\vec{B}$ is defined as the product of the magnitude of the vectors $\vec{A}$ and $\vec{B}$ and sine of the smaller angle between them.



$\vec{A} \times \vec{B} = AB\sin\theta\hat{n}$

- The cross product of the two vectors is at right angles to both the vectors and points in the direction in which a right-handed screw will advance.
- Properties of vector product:
  - The cross product of a vector with itself is a null vector.
  - The cross product of two vectors does not obey commutative law. That is,

    $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

  - The cross product of vectors obeys the distributive law. That is,

    $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

- If the vectors A→ and B→ represent the two adjacent sides of a parallelogram, the magnitude of cross product of A→ and B→ will represent the area of the parallelogram.